



M. F. Gilmullin, "Clever Soldiers at an Inn," *Mathematics with Mansur*, Apr. 9, 2025. [Online]. Available: <https://history-math.org/publications/hm-tt-a319495ed9d8-en/>.

PROBLEM

Clever Soldiers at an Inn

An old recreational problem by Ye. I. Ignatyev leads from soldiers' dinner bill to the Josephus recurrence.

Mansur Gilmullin · 09.04.2025 · EN

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Tilborch, Gillis van - Guardroom.jpg. The Guardroom by Gillis van Tilborch (c. 1625–c. 1678), from the Hermitage collection · [Source](#)

For your mathematical recreation, here is an old problem from Yemelyan Ignatyevich Ignatyev's book *In the Realm of Ingenuity*:

"Four tables stood in an inn, forming a quadrilateral. 21 hungry soldiers returning from manoeuvres stopped there for dinner and invited the innkeeper to join them. They sat as follows: the soldiers occupied three tables, 7 at each, while the innkeeper sat at the fourth."

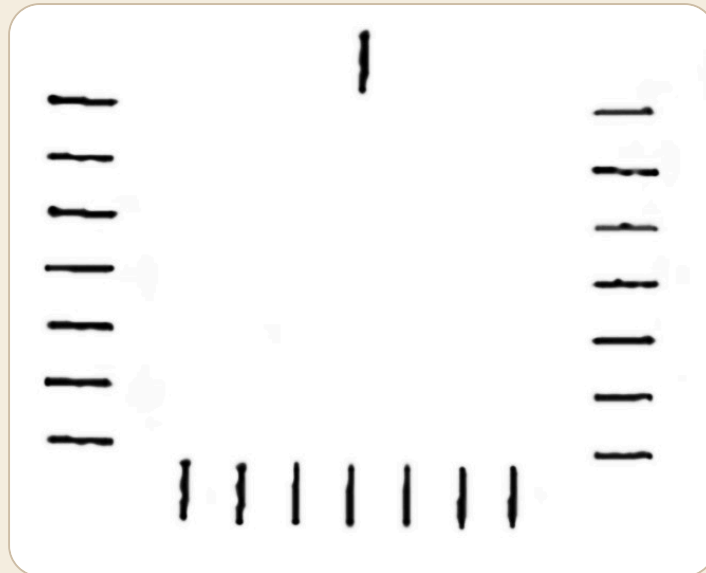


Diagram of the soldiers' seating for Yemelyan Ignatyevich's problem · [Source](#)

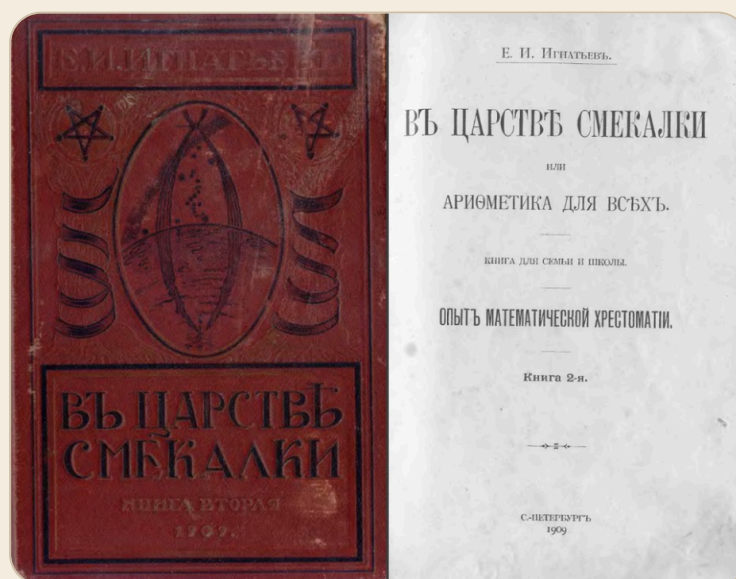
“The soldiers agreed with the innkeeper that the last person remaining would pay the bill under the following rule: count everyone, including the innkeeper, clockwise around the circle, and release every seventh person. Each person released left the inn, and the innkeeper himself was the last to remain. With whom did the counting begin?”

Ye. I. Ignatyev gives only the answer: *“The counting must begin with the 6th soldier seated to the innkeeper's left.”* He does not provide a solution method.

Try finding the starting person yourself in a similar case, for example if there were 4 soldiers at each of the three tables.

We will ask you to explain why the counting must begin there. The solution appears at the end of the article.

Historical background



Ignatyev, Ye. I. In the Realm of Ingenuity, or Arithmetic for Everyone. — Parts 1–3. — St Petersburg: V. L. Bogushevsky, 1908–1911 · [Source](#)

Yemelyan Ignatyevich Ignatyev (1869–1923) was a Russian and Soviet mathematics educator and author of popular-science books. His best-known work, *In the Realm of Ingenuity*, appeared in three volumes in 1908–1911:

Ignatyev, Ye. I. *In the Realm of Ingenuity, or Arithmetic for Everyone*. — Parts 1–3. — St Petersburg: V. L. Bogushevsky, 1908–1911.

The problem resembles the historical Josephus problem, known since 1612, when the French mathematician Bachet de Méziriac published it in his collection *Problem es Plaisants*. Its story is based on an account in Josephus' historical work *The Jewish War*.

1.3 ЗАДАЧА ИОСИФА ФЛАВИЯ

Последний из наших предварительных примеров представляет собой один из вариантов античной задачи, носивший имя Иосифа Флавия — известного историка первого века. Существует легенда, что Иосиф выжил и стал известным благодаря математической одаренности. В ходе иудейской войны он в составе отряда из 41 иудейского воина был захвачен римлянами в плену. Предпочитая самоубийство плену, воины решили выстроиться в круг и последовательно убивать каждого третьего из живых до тех пор, пока не останется ни одного человека. Однако Иосиф, начав с одним из своих кадиомышленников, счел подобный конец бессмысленным — он быстро вычислил спасительные места в порочном круге, на которые поставил себя и своего товарища.

В нашем варианте мы начнем с того, что выстроим в круг n человек, пронумерованных числами от 1 до n , и будем исключать каждого k -го из оставшихся до тех пор, пока не останется только один человек. Вот, к примеру, исходное расположение при $n = 10$:

Порядок исключения — 2, 4, 6, 8, 10, 3, 7, 1, 9, так что остается номер 5. Задача: определить номер указанного, $J(n)$.

Мы только что выяснили, что $J(10) = 5$. Можно было бы предположить, что $J(n) = n/2$ при четном n , тем более, что случай $n = 2$ подтверждает наше предположение: $J(2) = 1$. Однако некоторые другие частные случаи разочаровывают нас — предположение нарушается при $n = 4$ и $n = 6$.

n	1	2	3	4	5	6
$J(n)$	1	1	3	1	3	5

Придется вернуться к таблице и попробовать сделать лучшее предположение. Гм-м... Кажется, $J(n)$ всегда будет нечетно. И в самом деле, для этого имеется веское основание: первый обход по кругу исключает все четные номера. К тому же, если само n четно, мы приходим к ситуации, подобной той, с которой начали, за исключением того, что остается вдвое меньше людей и их номера меняются.

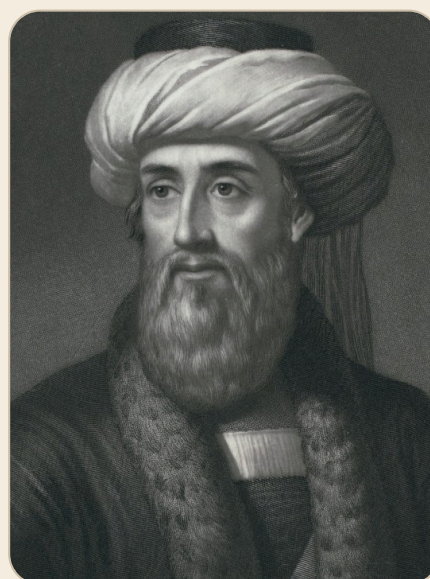
Вот случай, когда $n = 0$ не имеет смысла.

Даже медленная дотычка — не лучший вариант решения, поскольку она помешает выжить в задаче.

26 ВОЗВРАТНЫЕ ЗАДАЧИ

The historical Josephus problem in Knuth's *Concrete Mathematics: A Foundation for Computer Science* (2009) · [Source](#)

The problem is as follows: 41 soldiers stand in a circle. Starting with the first soldier, they kill every third person. Where should one stand to be the last survivor?



Imaginary portrait of Josephus. Illustration by Thomas Addis Emmet, 1880 · [Source](#)

Josephus (Latin Josephus Flavius, c. 37–c. 100 CE) was an ancient Jewish historian and military commander. He is known for works that survive in Greek: The Jewish War, about the revolt of 66–71, and Antiquities of the Jews, recounting Jewish history from the creation of the world to the Jewish War.

Solution

An algorithm for solving the problem appears in the book:

Donald Knuth, Ronald Graham, Oren Patashnik. Concrete Mathematics: A Foundation for Computer Science. [Translated from English.] — 2nd ed. — Moscow: Williams, 2009. — 784 pp.

This is a well-known recurrence problem in computer algebra. It is often used as a test problem in various settings, including interviews.

We will use the familiar recurrence formulas for the Josephus problem. In the more general version, n soldiers are counted around a circle and every k th soldier is killed. Denote the safe position, that of the last survivor, by $J(n, k)$.

Let us model the process by creating a list of 22 elements: 0 is the innkeeper and 1–21 are the soldiers.

The following recurrence is known in the language of modular arithmetic:

$$J(n, k) = (J(n - 1, k) + k) \bmod n, \text{ where } J(1, k) = 0.$$

In the classical Josephus problem with $n = 22$ and $k = 7$, the last survivor's position is calculated as $J(22, 7)$. Here we need the reverse: knowing that the last person is at position 0, the innkeeper, find the starting position. If the standard start at position 0 leaves the survivor at $J(n, k)$, then to leave position x last, counting must begin at $(x - J(n, k)) \bmod n$. The formula for the starting position is also known: $S = (x - J(n, k)) \bmod n$, where x is the desired last survivor.

For our problem, $n = 22$, $k = 7$, $x = 0$ (the innkeeper). Recursive calculation — try it by hand — gives $J(22, 7) = 16$.

$$\begin{aligned} J(1, 7) &= 0, \\ J(2, 7) &= (J(1) + 7) \bmod 2 = (0 + 7) \bmod 2 = 1, \\ J(3, 7) &= (1 + 7) \bmod 3 = 8 \bmod 3 = 2, \dots \\ J(21, 7) &= (2 + 7) \bmod 21 = 9 \bmod 21 = 9, \\ J(22, 7) &= (9 + 7) \bmod 22 = 16 \bmod 22 = 16. \end{aligned}$$

We need the last position to be 0. The formula therefore gives:

$$S = (0 - 16) \bmod 22 = (-16) \bmod 22 = 6$$

Thus, counting must begin at position 6.

Check: arrange all the participants in a circle — 0 is the innkeeper and 1–21 are the soldiers — and begin counting at position 6. Successively removing every 7th person leaves the innkeeper last.

Answer: counting must begin with the 6th soldier seated to the innkeeper's left.